



The Big Dinner

*Multiplication with
the Ratio Table*

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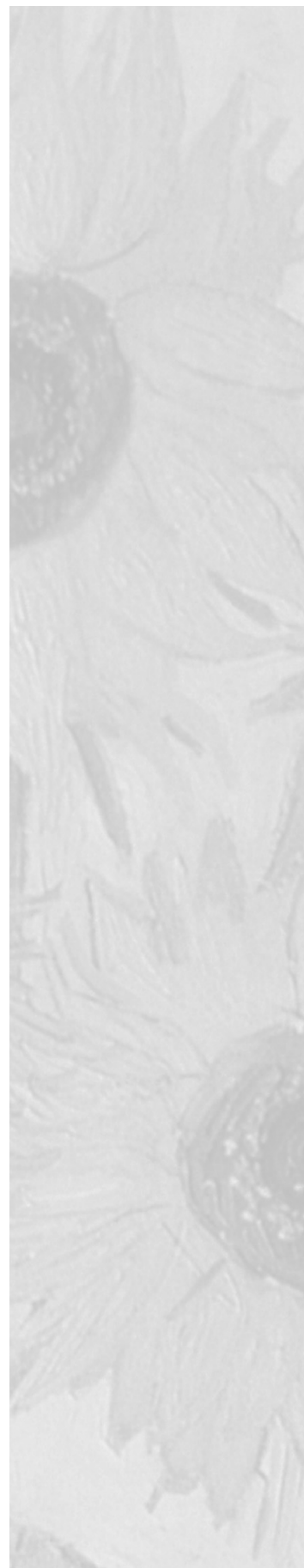
Lisa Cinelli

Schools featured in photographs

The Muscota New School/PS 314 (an empowerment school in Region 10), New York, NY
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Fort River Elementary School, Amherst, MA

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DAY ONE

Pricing the Ingredients

Today you will be developing the context, laying out the terrain for the work of the next two days. After you introduce the situation and show the posters of the big dinner and the turkey in the grocery store, students will set to work figuring out the cost of a 24-pound turkey. As you move around the room supporting, encouraging, and conferring, you will notice that students use a range of strategies to determine the total cost of the turkey.

Day One Outline

Developing the Context

- ☀ Tell the story of the big dinner.
- ☀ Ask students to figure out how much the 24-pound turkey will cost.

Supporting the Investigation

- ☀ Take note of the various strategies and struggles you see as students calculate the cost of the turkey.

Preparing for the Math Congress

- ☀ Ask students to make posters of their strategies.
- ☀ Begin to plan for a math congress that will support development of the use of more efficient strategies.

Materials Needed

The big dinner poster and the turkey poster

[If you do not have the full-color posters (available from Heinemann), you can use the smaller black-and-white versions in Appendixes A and B.]

Photocopies of the turkey poster (Appendix B)

—one per pair of students

Money—various amounts of quarters, dimes, nickels, and pennies

Drawing paper—a few sheets per pair of students

Large chart paper—one sheet per pair of students

Markers

Developing the Context

- Tell the story of the big dinner.
- Ask students to figure out how much the 24-pound turkey will cost.

Introduce the problem to the students by explaining that you will soon be cooking a big dinner for Thanksgiving (or a family party or a class dinner for parents). You plan to roast a turkey with potatoes and vegetables and make apple pie for dessert. Show the big dinner poster (or Appendix A). Continue the story by telling students that when you went to the grocery store to price turkeys, you saw a sign that turkeys cost \$1.25 per pound. Show the turkey poster (or Appendix B). Discuss what the students notice. Ask them to read the sign. Explain that you need to buy a big turkey to feed everyone and the biggest turkey in the store weighs 24 pounds. Invite students to figure out how much that turkey will cost.

Behind the Numbers

The numbers in this scenario were chosen to promote the use of a variety of grouping strategies and to support the development of partial products, using the distributive property. For example, students might use their knowledge of money and immediately realize that \$1.00 for each pound is \$24.00. Then they may use a variety of strategies to calculate the product of the 24 quarters. Others may think of adding the cost of four pounds together to get \$5.00 and then use this fact to calculate the total. While the numbers support these strategies, other students might begin with less efficient strategies (such as repeated addition). Efficient strategies should be encouraged and reflected upon as you confer with students.

Supporting the Investigation

Assign math partners and give each pair of students some blank paper and a photocopy of Appendix B for reference. Have coins available if students need them as tools. As students work, walk around and note the various struggles and strategies you see. Here are some possibilities:

- Take note of the various strategies and struggles you see as students calculate the cost of the turkey.
 - skip-counting by \$1.25
 - splitting the \$1.25 into \$1.00 and \$0.25 and then skip-counting or adding the 24 quarters, or grouping four quarters together to make a dollar. [See Figure 1] This strategy is a specific case of the distributive property: $24 \times \$1.25 = (24 \times \$1.00) + (24 \times 0.25)$.
 - using repeated addition

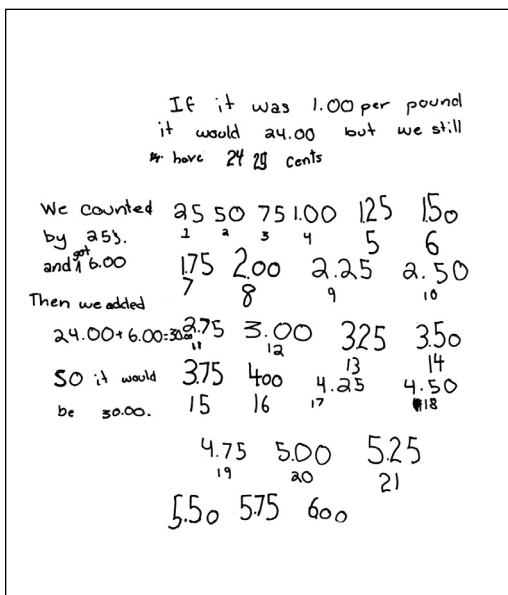


Figure 1

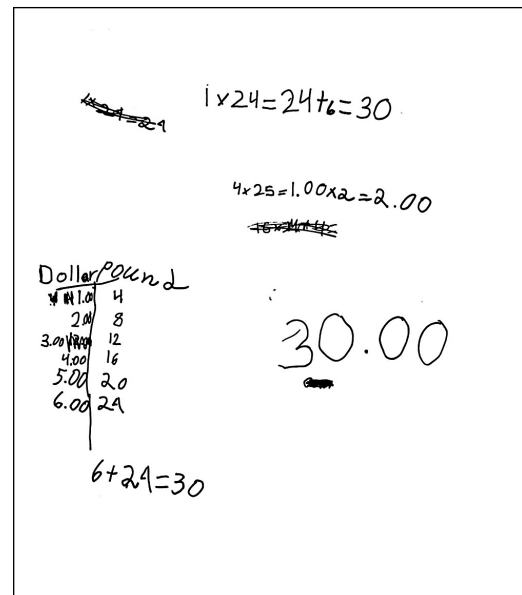


Figure 2

- ♦ using a ratio table. The ratio table is a helpful way to organize the multiplicative work in this context. In the sample work shown in Figure 2, page 14, the students organize the quarters: 4 pounds each costing \$0.25 would equal \$1.00; 8 pounds would equal \$2.00, and so on.
- ♦ using proportional reasoning—\$5.00 equals 4 pounds. In the sample work shown in Figure 3, the students use the number line as a tool to think with: they place 24 tallies on it to represent the number of pounds and draw \$5.00 as the leaps. On the bottom of the number line they keep track: \$5.00 for 4 pounds, \$10.00 for 8 pounds, etc.

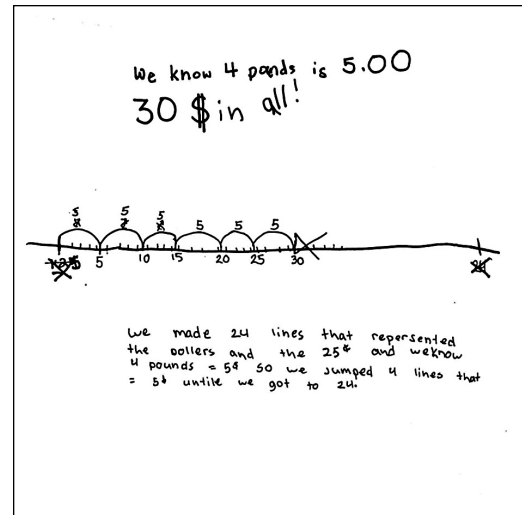
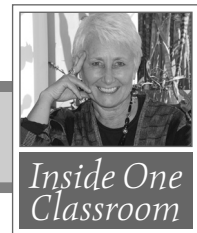


Figure 3

Confer as needed to support and challenge students. Do not show them what to do; instead, look for moments when you can encourage reflection and, if appropriate, puzzlement. Listen first and try to understand what the student is attempting to do. Supporting that idea, even if it is tedious, is the starting place of every conference. Then look for ways to encourage and challenge that will support development. Remember to work with the mathematician; don't just fix the mathematics!



Conferring with Students at Work

Diana (the teacher): How's it going?

Kari: This is hard. There are so many \$1.25s to add.

Pablo: We keep adding them and coming up with different answers.

Diana: Well, I wonder if there is a way you can keep track of what you're adding. You have so many \$1.25s to add up. How many do you need to add?

Pablo: Twenty-four, because there are 24 pounds.

Diana: What about labeling each of the \$1.25s you have here to make sure you have all 24 of them? Like this. (*Diana jots "1 pound" next to the first \$1.25.*) Could you do that? Do you think it would help?

Kari: Maybe. And we could write *p* instead of *pound* because it will be faster.

Pablo: But it's still so hard to add them all up.

Diana: Well, do you know what two \$1.25s are?

Kari: (*Looks back at other work.*) Umm... \$2.50.

Diana: Well, what about four then?

continued on next page

Author's Notes

When students attempt skip-counting or repeated addition strategies, the numbers can seem large and unmanageable. Here Diana begins with the students' strategy and encourages an important idea: how to keep track.

Labeling the pounds and the money sets up the data in a way that may encourage the emergence of a ratio table.

By suggesting a way to combine the numbers that might make the work of repeated addition less cumbersome, Diana guides Kari and Pablo to use something they know—that \$1.25 plus \$1.25 equals \$2.50. By asking about combining the \$2.50 plus \$2.50, Diana helps Pablo get to five, a friendly number that is familiar to him in skip-counting.

continued from previous page

Pablo: That would be \$2.50 plus \$2.50. Hmm...\$5.00.

Diana: Well, \$5.00 is a lot easier to add than \$1.25. I wonder if you could do that to figure it out. Put four together to make \$5.00?

Kari: Yeah, I think so.

Diana: Keep working on it and I'll come back to see how you're doing.

Since this pair of students now has a strategy they feel comfortable with, Diana leaves to confer with another group.

Preparing for the Math Congress

- ☀ Ask students to make posters of their strategies.
- ☀ Begin to plan for a math congress that will support development of the use of more efficient strategies.

As students finish their investigations, ask them to make large posters of their strategies for display. The purpose of making posters is to foster further reflection. Making posters challenges students to solidify their ideas. Deciding what important ideas they want to explain, what is extraneous, how to revise their draft work, and how to represent the main issues provides time for thinking about their own thinking: metacognition. Encourage students to display their strategies clearly on their posters so classmates will be able to read and understand what they did and their justification for why their strategies work.

As the students prepare their posters, take note of the different strategies you see. Look for two or three posters that would be useful focal points of discussion for the math congress scheduled for Day Two. The purpose of the congress is to support students in considering a variety of strategies, exploring how those strategies are connected, and examining them for efficiency. Plan on structuring the congress in a way that will potentially scaffold development toward the use of more efficient strategies. Also, ensure that the congress includes a discussion of helpful ways to keep track.



Reflections on the Day

Moving around the room and conferring with students as they worked gave you the opportunity to notice how the students were thinking about the work and what struggles they were having. Did any students recognize that they could split the \$1.25 into \$1.00 and \$0.25? Did students who used repeated addition realize that they could double the \$1.25 as a way to make the numbers easier to add? Did many students recognize the \$0.25 as a quarter and that four quarters equal one dollar? Did students find ways to organize their work or did they struggle to find a way to keep track of it all? Are there connections across student work in the ways they organized? These are all important observations to keep in mind as you prepare for the math congress on Day Two.

DAY TWO

Buying the Turkey

Today begins with a “gallery walk” during which students examine and post comments on each other’s posters from Day One. A math congress then convenes to discuss a few of the strategies and to consider helpful ways to make the problem easier and to keep track. The congress gives you a chance to focus on a big idea or strategy in the students’ work that you think will be helpful for your students to discuss.

Day Two Outline

Preparing for the Math Congress (continued from Day One)

- ☀ Conduct a gallery walk for students to review and comment on each other’s posters.
- ☀ Choose two or three posters to be discussed during the math congress.

Facilitating the Math Congress

- ☀ Facilitate discussion of the posters you have chosen.

Materials Needed

Students’ posters from Day One

Sticky notes—one pad per student

Markers

Preparing for the Math Congress (continued from Day One)

- ☀ Conduct a gallery walk for students to review and comment on each other's posters.
- ☀ Choose two or three posters to be discussed during the math congress.

Explain to the students that the class will have a gallery walk to look at each other's posters before starting a congress. Pass out small pads of sticky notes and suggest that students use them to write comments or questions that can then be placed directly onto the posters. Display all the posters around the room and give students about fifteen minutes to read and comment on the mathematics on the posters. You may need to model for your students how to comment in helpful, appropriate ways. You might write, "I think your strategy is interesting, and I'm trying to figure out why we have different answers." Or "You explained your work really well. I can understand what you did." Or "Your poster really convinces me. I agree with your thinking." Or "Your strategy is really great. What made you think of starting like that?" Or "Your way is so fast. I want to try that strategy next time!" Or "I'm puzzled about this step. I need more convincing." Give students a few additional minutes to read the comments and questions on their own posters and then convene a math congress.

■ Tips for Structuring the Math Congress

Since the purpose of this first congress is to consider ways to keep track and ways to make the problem easier, you might want to choose two or three posters that can be used to scaffold the conversation. For example:

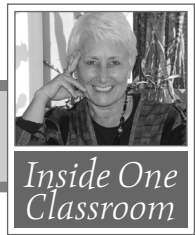
- ◆ Begin with an entry-level piece of work in which students skip-counted or used repeated addition, struggling with the large numbers.
- ◆ Then move to a poster showing that students isolated the dollars and grouped the quarters. Ask students to consider if breaking up the numbers is an easier way to work with the large numbers. Could such a strategy always be used? This discussion lays the terrain for the development of the distributive property as the unit progresses.
- ◆ If a group has used a ratio table to keep track of the number of pounds and the money simultaneously, you might want to focus conversation on this at the end of the congress. Starting with a piece of work like this would probably be too difficult, but scaffolding the conversation by discussing how the strategies are related is a good way to support development.

Facilitating the Math Congress

- ☀ Facilitate discussion of the posters you have chosen.

Gather the students in the meeting area and ask them to sit next to their partners. Begin by inviting one pair of students to display their poster and share their strategy.

A Portion of the Math Congress



Diana (the teacher): Shawn and Brianna, would you share your strategy with us?

Shawn: Well, we added \$1.25 twenty-four times.

Brianna: On the number line.

Diana: Put your hand up if you understand their strategy. Who can explain it?

Erika: I can. They used the number line and kept adding \$1.25 until they had added it 24 times.

Diana: Whew! That was a lot of work for you two!

Brianna: It was. It took us a long time and we got messed up a few times.

Diana: How many other people did it this way? (*A few hands go up.*) It was a lot of work, wasn't it? When we walked around looking at the posters, I thought maybe it would help to talk together about some ways to make the problem friendlier. You had an interesting way to make it easier, Rosa and Khalil. Help us understand your way.

Rosa: We knew that \$1.00, 24 times, would be \$24.00. So we just worked with the quarters. Four quarters is a dollar, so we drew 24 quarters and circled 4 of them at a time. That made \$6.00.

Diana: Turn to the person next to you and talk about this strategy. What did they do? Was it easier? (*After a few minutes of pair talk, Diana resumes the whole-group conversation.*)

Brianna: Ohhhh... I think they only had to do the quarters because they knew that the dollars would be \$24.00.

Rosa: Yes, that's what we did. Then we just circled all the quarters.

Anna: Oh, that is so much easier.

Susannah: We did that, too, but instead of circling the quarters...

Diana: Come up. Show us.

Susannah: We kept track of the quarters like this. One dollar was 4 pounds; 2 dollars was 8 pounds.

continued on next page

Author's Notes

Diana begins by recognizing the hard work many students have done. By establishing repeated addition as a possible strategy, she invites all the students to consider this as a possible way to mathematize the situation. To ensure that everyone has a way to enter this conversation, she checks to see if everyone understands by asking who can explain the strategy. Many teachers ask: Does everyone get it? Students will often nod in response even if they don't really understand. By asking instead who can explain, Diana can see who needs further clarification, since only those students who can explain the method will raise their hands.

In an attempt to foster development, Diana helps the students (and the community) to own a new mathematical question: How can we make this problem friendlier?

Rosa and Khalil used the distributive property. They made two partial products: $24 \times \$1.00$ and 24 quarters. Diana engages the students in pair talk here to heighten reflection on this strategy. Pair talk at critical moments like this implicitly says to the community: this is an important math idea for us to think about; it is important for us to try to understand each other's methods and to discuss them.

Diana invites the students to explain. She does not explain for them. Like mathematicians, students must represent their ideas, communicate them, and convince the community that their ideas and strategies work.

Diana: Stop there for a minute. Let's see if people are following you.
(Several people paraphrase, and then Susannah finishes explaining.) Pablo, you and Kari had a neat way, too. Tell us what you did.

Pablo: Our way is sort of like theirs, but we doubled the \$1.25 because we saw that \$1.25 plus \$1.25 equals \$2.50. Then we did 2 of those. That was \$5.00. So we knew 4 pounds was \$5.00. We kept doing that. Then we just had to add the fives. That was easy, too.

Diana: Wow! So it seems we have a lot of ways to make the work easier. Mathematicians like doing that—looking for nice friendly ways to figure out a problem. So let's take a look at these four posters together. Let's put them next to each other and see if we can talk about how they are related.

The relationship among all these strategies is the distributive property, or making partial products. Groups of one (repeated addition) work, but are tedious to use. Taking two groups of \$1.25 or four groups of \$1.25 at once makes the work easier. Students can also group by splitting the \$1.25 into dollars and quarters. By placing the posters together and asking the class to consider how they are related, Diana is encouraging the students to realize this property and to generalize that, as long as they are sure they have all the pieces, they can break up the numbers in lots of ways.

■ Assessment Tips

It would be nice to place the posters in students' portfolios; however, they are probably too large. If so, you can take a photograph of each poster and staple it to a blank page for your anecdotal notes. Make notes about the strategies and big ideas described in the overview of this unit (pages 6–8). Do you have evidence that any of these ideas and strategies have been constructed? The work completed today is evidence of students' ideas and strategies at the start of this unit.



Reflections on the Day

In the gallery walk today students read and commented on each other's mathematics, like professional mathematicians. Doing so gave them a chance to reflect on a variety of strategies. Discussion in the math congress deepened these initial reflections by focusing on the efficiency of grouping strategies. As the unit progresses and students work toward generalizing distributivity, associativity, and the uses of the ratio table, these grouping strategies will be supported and further encouraged.