Investigating

Multiplication and Division

Unit Overviews for Grades 3–5

Investigating Multiplication and Division (Grades 3–5) is organized around 5 units. Each unit is developed around carefully-crafted contexts—realistic and fictional— and comprises a two-week (10-day) sequence of investigations, games, routines, and minilessons.

1 Groceries, Stamps, and Measuring Strips: Early Multiplication

BY FRANS VAN GALEN AND CATHERINE TWOMEY FOSNOT

The focus of this unit is the introduction and early development of multiplication. By making use of realistic contexts, the unit invites students to find ways to mathematize their lived worlds with grouping structures. The unit uses many contexts: inside the grocery store; postage stamps; city buildings, windows, and buses; tiled patios; a baker's trays; and sticker pages. Initially, formal multiplication notation is not the focus; efficient grouping is, as students are encouraged to make groups (and groups of groups) to find efficient ways to deal with repeated addition and determine totals.

The unit begins with the context of a grocery store. Students view an illustration of fruits and vegetables arranged in bins, stacked packages of paper towels, and six-packs of water bottles, among other items in a grocery store. Although the objects shown can be counted by ones, the arrangements naturally invite repeated addition, skip-counting, and doubling strategies as well as the language of grouping—for example, 8 groups of 6 is equivalent to 4 groups of 12 which is equivalent to 4 groups of 6 plus 4 groups of 6.

The stamp context used next eliminates objects that can be counted by ones. Now the value printed on the stamp is the focus. This context thus supports the development of unitizing by providing the value (e.g., seven cents) as a unit that can be counted. Providing the value also offers a built-in-constraint to counting by ones, and supports repeated addition and efficient grouping employing doubling, doubling and halving, and the addition of partial products. This context promotes a natural shift in students' language to "5 sevens" (5 seven-cent stamps).

Formal notation (the use of x to indicate multiplication) is introduced halfway through the unit with the context of measurement. Students view an illustration of a cityscape with high buildings, large windows, tall trees, and a school bus. A four-foot tall, eight-year-old boy, Antonio, is shown on the street; Antonio wonders how much taller everything is than he. Because his height is used for a unit of measurement, the natural language that evolves is "times, for example, "eight times the size of Antonio". Formal notation is introduced to match the language—8 x 4.

As the unit progresses, students make a set of measurement strips (for the multiplication tables) and explore the relationships between the products on them. In the last few days of the unit, the measurement strips are represented as number lines that students use to determine missing products from the expressions and products provided. Here the five- and ten-structures are emphasized, supporting students in using five-times to help with four-times and six-times, and ten-times to help with nine-times.

Several minilessons are also included in this unit. Quick images, count-around-the-circle activities, and pictures with built-in constraints support the construction of efficient strategies—strategies that over time will help students automatize the basic facts.

Note: This unit also incorporates aspects of the measurement strand as students measure the height of various objects in the illustration of the city. They use the height of Antonio as an iterated unit, and make measurement strips for the lengths of various groups of connecting cubes.

2 The Big Dinner: Multiplication with the Ratio Table

BY CATHERINE TWOMEY FOSNOT

The focus of this unit is the development of multiplication, including automatizing the facts, using the ratio table, and developing the distributive property with large numbers. The unit begins with the context of the preparation for a big turkey dinner. This story context sets the stage for a series of investigations. First, students investigate the cost of a 24-pound turkey, selling for $1.25 per pound. As the unit progresses, students develop t-charts for the grocer that list the prices of turkeys of various sizes and of various amounts of apples, carrots, and potatoes. Students then use these charts to calculate the total cost of all the ingredients, working with the ratio table and the distributive property. The unit culminates in an investigation to determine how long the turkey should cook, at fifteen minutes per pound.

Several minilessons for multiplication are also included in the unit.These use strings of related problems as a way to explicitly guide learners toward computational fluency with whole-number multiplication and to build automaticity with multiplication facts by focusing on relationships.

Note: This unit also incorporates aspects of the measurement strand as students calculate the cost and cooking time of the turkey (and other ingredients). If your students have not had prior experience with measuring weight and time, you should do several measurement activities first. Such activities might include weighing a variety of things in pound units and determining how many minutes there are in an hour (ensuring that students understand that 15 minutes is a quarter of an hour and 30 minutes is a half).

3 Muffle's Truffles: Multiplication and Division with the Array

BY ANTONIA CAMERON AND CATHERINE TWOMEY FOSNOT

The focus of this unit is the development of the open array as a model for multiplication and division.This unit uses a series of investigations based on the context of Muffles' Truffles shop. The questions posed in the first investigation (how many boxes of ten can be made with a given quantity of truffles; how many leftovers will there be from a given quantity and how can they be combined to make assortment boxes; and what is the cost of a given quantity of truffles if they cost $1 each) give students an opportunity to explore place value—the multiplicative structure of our base-ten system and quotative division. In the second and third investigations, students build two-dimensional blueprints of one-layer boxes and use these arrays to explore some of the big ideas in multiplication (the distributive, associative, and commutative properties). In the fourth and final investigation, students work with open arrays in the context of labeling and pricing wrapped boxes of truffles.To figure out the dimensions of the wrapped boxes (or open arrays) and the cost, students need to apply a number of big ideas previously developed in this unit.

There are three different kinds of minilessons for multiplication included in the unit as well: counting around the circle, strings of related problems, and quick images. The count-around is used to support the development of place value as it relates to multiplication. The strings of related problems are explicitly designed to guide learners toward computational fluency with whole-number multiplication and to build automaticity with multiplication facts by focusing on relationships. The quick images use 2 x 5 and 1 x 5 arrays as units to build larger arrays. In the last days of the unit, more complex minilessons (double-digit multiplication problems) generate a wider range of student strategies that can be explored (and modeled) with the open array.

4 The Teachers' Lounge: Place Value and Division

BY CHRIS NATALE AND CATHERINE TWOMEY FOSNOT

The focus of this unit is division. It begins with the story of a teacher noticing a service person in the teachers' lounge fill two different vending machines with beverages. In the first machine, there are bottles of water only. The machine holds 156 bottles of water when full and the teacher wonders how many six-packs that might be. The second problem involves the juice machine. It also holds 156 bottles when full, but the bottles are partitioned into six columns because there are six different flavors of juice. The teacher wonders how many there are of each flavor.

Although most students do not realize it at the start, the two problems are related. The first problem is a quotative division situation—the amount in each group is known, the number of groups is not. The second problem is a partitive division situation—the number of groups is known, the amount in each group is not. The problems are juxtaposed and given together to encourage students to examine the relationship between the two kinds of division.

This story context of the teachers' lounge sets the stage for a series of investigations designed to support the development of a repertoire of strategies for multiplication and division, including the use of:

- the ten-times strategy
- partial products and partial quotients
- the associative property
- the distributive property of multiplication over addition—the basis for the long division algorithm

Several minilessons for multiplication and division are also included in the unit. These are structured as strings of related problems designed to more explicitly guide learners toward computational fluency. Toward the end of the unit, discussion shifts to how the context of a division problem influences what to do with the remainder.

Note: The context for this unit assumes that your students will have had prior experience using arrays for multiplication. If this is not the case, you might find it helpful to use the unit *Muffle's Truffles* first.

5 The Box Factory: Extending Multiplication with the Array

BY MIKI JENSEN AND CATHERINE TWOMEY FOSNOT

The focus of this unit is the deepening and extending of students' understanding of multiplication, specifically the associative and commutative properties and their use with computation, systematic factoring, and the extension of students' understanding of two-dimensional rectangular arrays to three-dimensional arrays within rectangular prisms.

The unit includes a series of investigations based on the context of a cardboard box factory. Initially students design a variety of boxes (rectangular prisms) that hold 24 items arranged in rows, columns, and layers. The questions posed in the first investigation (how many box arrangements are possible, and how do we know for certain that we have found all the possibilities) give students an opportunity to explore the associative and commutative properties, factor pairs, doubling and halving strategies, and systematic ways of organizing their work to determine all possible cases. In the second and third investigations, students analyze the amount and cost of the cardboard needed for their boxes, deepening their understanding of the associative property, examining congruency vs. equivalency, and exploring the relationship of surface area to the shape of the box. Subsequent investigations involve using two different cubic boxes as units of measurement, and determining the volume of a shipping box that measures 4 feet by 6 feet by 4 feet. By the end of the unit, formulas for surface area and volume of rectangular prisms are the focus.

Several minilessons for multiplication are included in the unit as well—these are structures as strings of related problems explicitly designed to guide learners toward computational fluency with whole-number multiplication, by focusing on factors and efficient grouping.