Investigating
Fractions,
Decimals,
and Percents
Grades 4–6

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From Exploring Parks and Playgrounds

Markers at every 1/12
Water at every 1/8

START

FINISH
Teaching and Learning in a Math Workshop

Overview and Description

The Contexts for Learning Mathematics units assume a workshop structure in the classroom. You may be more familiar with a workshop model for literacy than for math. There, children write, revise, share their work with others, and celebrate and publish their stories. Teachers move around the room and confer with young writers, questioning, challenging, and supporting. Children have work folders for pieces in progress. Portfolios are used for selected pieces that document the students’ progress as writers.

Math workshops are very similar. Philosophically they are based on the same learning theory—the belief that knowledge emerges in a community of activity, discourse, and reflection. We learn to write by writing and discussing our writing with other writers. Similarly, we become mathematicians by engaging with mathematical problems, finding ways to mathematize them, and defending our thinking in a mathematical community.

When classrooms are workshops, learners (no matter how young) are inquiring, investigating, discussing, and constructing. They put forth their mathematical ideas in the community of their peers and justify and defend their thinking. Teachers encourage students to explore, notice patterns, develop efficient strategies, and generalize ideas. In a very real sense, when classrooms are turned into workshops children can become young mathematicians at work (Fosnot and Dolk 2001a, 2001b, 2002; Cobb 2005; Schifter and Fosnot 1993).

The heart of the math workshop consists of ongoing investigations developed within contexts and situations that enable children to mathematize their lives. As children work, the teacher moves around the classroom, listening, conferring, supporting, challenging, and celebrating. After their investigation, children write up their strategies and solutions and the community convenes for a math congress. This is more than simply a whole-group share. The math congress continues the work of helping children become mathematicians in a mathematics community—it is a forum in which children communicate their ideas, solutions, problems, proofs, and conjectures to each other.

Out of the congress come ideas and strategies that form the emerging discipline of mathematics in the classroom (Cobb 2005). Norms get established: What holds up as a proof, as a convincing argument? What counts as a beautiful idea or an efficient strategy? What does it mean to talk about mathematics and to symbolize it? What makes a good question, one
worth pursuing? (Yackel 2001). All the answers to these questions emerge in the interactions and discussions you facilitate. Throughout the units, sections titled “Inside One Classroom” highlight sample dialogue and supporting commentary to help you envision interactions during conferences, as children work, and in subsequent math congresses when they present and defend their work.

There are five components to the flow of a good math workshop:

- developing the context
- supporting the investigation
- preparing for the math congress
- facilitating the math congress
- integrating minilessons, games, and routines

All five components may not occur every day because some investigations span several days. Many teachers therefore find it helpful to think about the components separately as a way to describe the flow of planning.

### Developing the Context

Contexts for investigations are most frequently developed with stories and pictures that are carefully crafted to support the development of big ideas, strategies, and models. Developing the context with your students is like setting the stage or laying out a terrain that will intrigue children and ignite their imagination. Good contexts are situations—either realistic or fictional—that students can imagine, that enable them to realize and reflect on what they are doing, and that will potentially have an effect on mathematical development. Within these rich, imaginative contexts children can make sense of the strategies they try out, explore and generate patterns, generalize, and develop the ability to mathematize their own lived worlds. The context is usually developed in a meeting area with the whole class. It is often helpful to have a large easel with chart paper and/or a large chalkboard or whiteboard nearby.

### Supporting the Investigation

After a context is developed, the teacher usually assigns math partners and the children set off in these pairs to investigate. It is important to think about ways to design the classroom so that workshop areas are readily available and conducive to discussion and exploration. Tables work best. Needed materials should be handy and available to the children so they can work autonomously without being dependent on you for every little need. If some children get distracted easily and find it difficult to get started, you might want to ensure that they are settled first and have a way to get started. Then begin to move around the room, observing the strategies that children use...
and listening to their discussions. Confer with some children. If you feel it would be helpful to focus reflection or increase the challenge, ask questions. The best questions and comments are designed not to lead children to your answers, but to encourage them to reflect on what they are doing:

- “That’s an interesting way to begin.”
- “Help me understand your way.”
- “What made you decide to start like that?”
- “What will you do next?”

You might want to see if both partners can explain what they are doing and have them ask each other questions if they cannot. Keep students grounded in the context if they don’t know how to start. For example, when working with the unit *Field Trips and Fund-Raisers*, one teacher commented, “I’m puzzled, too! I’m wondering how we could compare these portions. If you build the sandwiches with these cubes, how long do you want them to be? You want them all to be the same length, right? So it would be fair?” She did not say, “How can you compare 3⁄4, 4⁄5, 7⁄8, and 3⁄5? Is there a common denominator you can use?” Instead, she asked about lengths of subs—she stayed grounded in the context.

### Preparing for the Math Congress

Everybody needs time to prepare for the math congress—you and your students. Help the children prepare by asking them to talk with their partners about what they want to share. Have them make a poster of the ideas or strategies they want to share and discuss. You might also encourage them to walk around and look at each other’s posters during a “gallery walk.” They can write questions and comments on sticky notes and place them on the posters. This activity engages children in reading and commenting on each other’s mathematics.

As children are preparing for the congress, prepare yourself. Note carefully the various strategies and ideas students are using so that you can facilitate a powerful discussion. Imagine how the conversation in the congress should flow:

- What ideas deserve discussion? In what order?
- Can some of the ideas be generalized? How will you promote this?
- Is there a possible sequence in the discussion that might serve as a scaffold to learning?

How you structure the discussion is very important. For that reason, tips on how to structure congresses are provided throughout the units. Whatever structure you decide on should support the development of the mathematicians in your community. *Don’t try to fix the mathematics; work with the mathematician.* The point is not to fix the mistakes in the children’s work or to get everyone to agree with your answer, but to support your
students’ development as mathematicians. Challenge them to think. Ask them to reflect on inconsistencies and answers that aren’t reasonable. Invite them to inquire further. Wonder with them about appearing patterns. Model the joy of mathematical inquiry.

**Facilitating the Math Congress**

Math congress is not just a whole-class share. First of all, there is not time for everyone to share. Second, many of the children’s strategies will be similar; sharing them all would only be redundant. Third, powerful math congresses are structured to push the mathematical development of your community. You need to carefully think out which pieces of student work to use and the order of presentation; you might select only two or three pairs of students to present their work. One possible structure is to begin with a strategy that is inefficient but easy for everyone to understand in order to provide an entry level into the discussion for all. Choosing progressively more efficient strategies next can provide a challenge for the group and an invitation to consider how work can be made more efficient. Another possible structure is to choose pieces of work that are related around a specific big idea. As the work is presented, focus the community discussion on the idea and push for generalization with questions like these:

- Do you agree that this strategy will always work?
- Why is this so?
- Could we prove it?
- When is it helpful? When not?

Yet a third structure might be based on the representations children have used. Which are helpful? Which over time can become generalizable models as tools to think with? The landscape of learning can be a helpful tool as you consider which pieces of work to use to support the development of your community, and, as mentioned earlier, tips are provided throughout the units to help you facilitate powerful discussions.

Often teachers make the mistake of using a piece of work that they want shown so everyone else will use that strategy. If mathematical thinking were a simple case of transmitting ideas, why go through all of this inquiry? Why not just show and tell students the strategy you want them to use and have them practice it? The simple truth is that learning is much more complex. Sharing that strategy will help only if most of the community is at a place developmentally where the strategy will make sense to them. Any decision you make should be conducive to development. For that reason, it can be helpful to have a student share who has struggled with an idea, rather than one who has come by it easily. The process of sharing and discussing the idea, attempting to explain it to the community, can strengthen and clarify the understanding of the speaker.
Throughout the units, the sample dialogues called “Inside One Classroom” will help you envision how the conversation might flow. These sections provide a window into one teacher’s math congress. They are not scripts to follow, only sample dialogues that occurred.

**Integrating Minilessons, Games, and Routines**

At the start of math workshop, teachers often take ten or fifteen minutes to conduct a minilesson to highlight a computational strategy, share a problem-solving approach, or do mental math work. In a minilesson, teachers may take an explicit role in bringing up ideas and strategies for discussion, but the ideas are put forth only for consideration and examination. Games and routines also have a place in math workshop. Games can be very powerful if they are developed in ways that support new strategies and the discussion of big ideas (rather than just for practice and reinforcement). The same point can be made about routines such as taking attendance, preparing for snacks, and distributing materials.

**Time Frame**

Except for the resource units, each unit includes ten days of plans. It is assumed that the math period in grades 4–6 is 60–90 minutes. Sometimes most of the period is used for a minilesson and the development of the context for a new investigation, with the children working with a partner on the investigation for the remainder of the time. In some cases, a math congress ensues on the same day, but often it follows on the next day to allow children sufficient time to reflect on their work, make posters, and decide on the main ideas they want to prove and discuss in math congress. To help you with the flow of planning for a specific investigation, sections are marked throughout the unit as Developing the Context, Supporting the Investigation, Preparing for the Math Congress, Facilitating the Math Congress, and Minilesson.

The following pages, taken from Day One of *The Mystery of the Meter* unit, have been annotated to highlight the five workshop components, as well as the key features that will help facilitate your use of the materials.
The contexts in *Investigating Fractions, Decimals, and Percents* are created through carefully-crafted *posters*. These images are also provided in a reproducible format in the appendix of each unit.

The **concise outline of the day’s teaching moves** is an ideal guide to reference as you teach.

**Materials Needed** lists all of the resources you and your students will use during the workshop.

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**The Weird Dials**

Today you will tell the story of a boy named Zig who finds five weird dials on the side of his house. These dials form the context for the investigations in the unit. Although he does not yet know that the dials are part of an electric meter, Zig investigates their mathematical properties. Like Zig, the students in your class study the relationships between the dials as their hands turn. These initial inquiries will lay the foundation for the big ideas regarding equivalence, place value, and powers of 10 that the students need for later work with decimals.

**Day One Outline**

**Developing the Context**
- Introduce the weird dials context and have students talk in pairs about what they notice about the dials and what they think the dials show.
- Convene a whole-group discussion of the dials, and record students’ observations on chart paper.
- Distribute Appendix C and invite students to investigate the dials.

**Supporting the Investigation**
- Students need to construct for themselves how the motions of the hands on the dials are related, so it is important not to tell them how to read the dials or explain to them about decimal notation.
- Encourage students to consider the differences in the numbers and remind them that each recording was done ten minutes apart.

**Preparing for the Math Congress**
- Ask students to make posters explaining what numbers they think the dials indicate and the rationale for their thinking.
- Plan for a congress discussion that will focus on the relationships among the dials.

**Facilitating the Math Congress**
- To encourage consideration of how the dials are related, scaffold a discussion that will culminate with students comparing parts of rotations on one dial to whole rotations of another dial.
The context for every unit's investigation is carefully crafted to support the development of the big ideas, strategies, and models. It sets the stage for learning in a way that will intrigue children and ignite their imaginations.

A series of bulleted notes strategically placed in the side column help you navigate through each day. These highlight the key teaching moves you will want to attend to during each stage of the math workshop.

The main column contains step-by-step teaching advice, professional insights, and detailed suggestions for supporting and extending student learning.
The **Inside One Classroom** feature offers a glimpse into a classroom community of teachers and students as they explore the mathematics of the unit.

The **dialogue excerpts** model teaching language and are designed to help you envision interactions during minilessons, investigations, and math congresses.

The side column contains **notes**—my own professional insights on the dialogue and interaction.
The units in *Contexts for Learning* have been designed for maximum effectiveness. For example, the numbers are carefully chosen to represent landmark numbers or number relationships that are especially telling. **Behind the numbers,** explains the significance of the numbers chosen, why they are ordered the way they are, and how they work with the context to support the development of certain strategies, models, and big ideas.

### Behind the Numbers

The numbers in this beginning investigation have been selected to highlight certain relationships. They are 35, 96, 35, 96, 36, 101, 36, 106, and 36, 06. The values indicated by the dials increase by a constant amount, by 0.025 over each ten-minute period. Do not expect students to consider this as $\frac{5}{25}$ and do not push them to use decimal language. Just encourage them to describe the changes of the hands on the dials. For example, over the first ten minutes, the hundreds hand seems to move up from near the 6 to half way between the 6 and 9. Let students count the numbers between the first and second locations of the hand on each dial. Students may think that these dials are part of a large clock, and this theory is quite reasonable at this point. It is also a useful theory because it gets students to focus on the relationships among the dials.

As students discover that one rotation of a dial causes the value on the dial on its left to increase by one number, or one-tenth of a rotation, they are grappling with two big ideas that are necessary for their understanding of decimal fractions: the ideas of part-whole relations and equivalence. Their understanding of the initial big ideas will guide them to increasing facility with decimals as they work through this unit.

Another primary goal of this initial investigation is to enable students to construct language through which they can discuss the dials. This vocabulary builds upon the chart began in the opening discussion. More observations can be recorded on the chart.

* Students need to construct for themselves how the rotations of the hands on the dials are related, so it is important not to tell them how to read the dials or explain to them about decimal notation.
* Encourage students to consider the differences in the numbers and remind them that each recording was done ten minutes apart.

### Supporting the Investigation

In this investigation, students will grapple with the key ideas embedded in the context: that the rates at which hands turn are related by multiples of ten and that this understanding can be used to determine what the proper reading of the meter is. In effect, the meter is like one clock with five hands (except with separate dials for each hand and with the directions of rotation alternating). To read the meter, one records the highest digit completely passed by the hand on each dial (again being careful about the direction of rotation). But we do not directly teach this in this investigation because we first want students to construct the ideas that underlie the relationships of the dials. By constructing how the dials are related first, students will have a more robust understanding when they begin to work with decimals later in the unit.

Students may need to be reminded that each row of dials in Appendix C is Zig’s recording and that the recordings were made ten minutes apart. The time is important. The context of this investigation is structured so that students will see how the dials increase at regular time intervals and will therefore consider the relationships between the dials as they turn. As you move around and confer with students, encourage them to look at the differences and remind them that each recording was made ten minutes apart.

The **heart of math workshop** consists of ongoing investigations developed within contexts and situations that enable children to mathematize their lives. As children work, the teacher moves around the classroom, listening, conferring, supporting, challenging, and celebrating.
Recording sheets and other teaching tools are provided in a reproducible format at the end of the respective unit. They are also provided in an easy-to-access PDF format on the Teaching Resources CD-ROM.

Note how the questions on the recording sheet are formulated:

+ How do you think the hands on the dials move?
+ If you were to write down numbers for what you see, what would you write?

The questions have been formulated in this way because at this point the students should not be instructed on how to read the dials, nor is this the time to explain decimal notation. The objective here is for students to see that a hand on the right moves ten times faster than the hand on its immediate left and to use this idea to determine how to read the dials. In the days that follow students will construct an understanding of how decimals work, but in order to make sense of them they need to first understand how the motions of the hands on the dials are related. As they record a sequence of numbers for what they believe the dials read, have them insert a period (decimal point) between the ones and the tenths numerals. This is social knowledge: students won’t necessarily do this without your pointing out that the period is written between those two dials as part of the recording, and therefore it is a convention they should all follow. Have them use it because it is written there, but don’t try to explain decimals! As you move around and confer, here are some strategies you can expect to see:

- Recording the numbers on each dial as a way to determine the change—for example, realizing that in Zig’s second reading the hand on the last dial (the thousandths) has moved 5 (from 1 to 6) and the hand on the next dial (the hundredths) has moved 2 (from 6 to 8).
- Noticing the tick marks between the numbers and attempting to determine the relationship they have to the movement of the hands on the dials to the right. For example, a student might believe that when the first set of dials reads 35.981, the second dial in that sequence reads 5 3/8. The tick marks show the tenths of the unit of the dial. A reading of 35.900 could also be read as 3 90/100, 5 3/4, or 3 9 tenths, 0 hundredths, 0 thousandths (if a student believes that each dial is recorded separately and also looks at the tick marks and records them as well). As you confer with students, support them in examining the relationships among the dials. They will learn to record one number for the meter shortly.
- Deciding to write the numbers in a column, like this:
  35.961
  35.986
  36.011
  36.026
  36.061

Students might then notice the sequence of the digits appearing vertically, and attempt to continue the sequence in order to examine what happens with the hands on the second and third dials—when will the hand pointing to 0 on the third dial point to 1 and when will the hand pointing to the 7 on the second dial point to 7? Provide students with pictures of blank meters (Appendix D) so they can continue the sequence if they wish.
Conferring with Students at Work

Maria: The first row of digits is 3 6 0 6 1.

Lucy: I'm not sure. Why is the third number a 0 and not a 9? It's in between the two numbers.

Maria: Yeah, but then we don't know what any of them are. If we don't call it zero, what do we call it? What do we do? Can you help us?

John (the teacher): Maybe you should talk about how you think the hands move.

Maria: You mean like if they were a clock?

John: That's a good start.

Lucy: But in a clock there are two hands but the hour hand goes a lot slower than the others.

John: Think about that. Are the hands ever in between numbers? I'll check back with you a lot later.

(John moves on to confer with another pair of students.)

Alain: (Addresses John) We're talking about all the little lines between the numbers. They must do something.

Toni: Like they're there to confuse us.

Alain: No, really. I think there are two numbers on each clock. They just didn't put all the lines on.

Toni: Oh, I get it, the first row of clocks made, let's see, 35, 59, 96, 61, and 10. That's a lot of time zones.

John: It seems that some of the digits are repeating.

Alain: Yeah. The last number is the first.

John: That's interesting. Where was the hand pointing on the second dial when it was 9 on the third dial?

Alain: It was almost on the 6... like on the little ninth mark... vs. 9 out of 10 of the little marks.

John: And then when the third dial went to 0 and the second one went to 0? (referring to the third row, 36 01?)

Toni: Oh... now it is right on the 6... just about.

continued on next page
Preparing for the Math Congress offers strategies for helping your students organize and present their findings and tips on how to orchestrate and scaffold powerful discussions.

The numbers indicated on the dials increase by equal amounts every ten minutes. As students record they may not realize this, especially because of the ambiguity of what to record when dial hands are between numbers. Many students will select the number closest to the marker. Listen for conversations about this choice—these may be good cues for groups to share in the math congress. If students always choose the closest number, don’t press them to change during the inquiry; instead, make a note to be sure they raise this issue during the congress. Sorting this out is crucial for their development.

Preparing for the Math Congress

Distribute a sheet of large chart paper to each pair of students and ask them to prepare a poster listing the numbers that they believe the dials indicate and explaining why they read the meter in this way. Rather than having students draw figures of meters repeatedly, provide students with multiple copies of the blank meters (Appendix D) so they can concentrate on placing hands on the dials. As students work on their posters, think about how you will structure the congress.

Tips for Structuring the Math Congress

It is helpful to choose two or three pairs of students to share during the congress. Ask them to share their work one at a time and use the work as the focus of discussion. Keep in mind that the purpose of this math congress is for students to have a conversation about the movements of the hands on the dials—the relationships between the dials. Some students may have correct numerical recordings but may not be able to explain why they chose the lower number when the arrow was between numbers. Such a pair might be good to start with because you can prompt conversation about the "why" with the whole class. Those that disagree should be encouraged to speak up. Some groups may forget the rotation reversal and they can be heard as well. Another good choice as a way to begin is to compare two different readings for the same set of dials. This comparison may evoke a discussion of the tick marks and the equivalence, for example, $30\% = 3.9$. To deepen this understanding, next choose a pair whose work would initiate a discussion of the relationship between rotations of one dial, and how much the dial to the left of it rotates. This topic, with students comparing parts of rotations on one dial to whole rotations of another dial and why they are equivalent, is what the students should be discussing toward the end of the congress.
The math congress continues the work of helping children become mathematicians in a mathematics community—it is a forum in which children communicate their ideas, solutions, problems, proofs, and conjectures to one another. Out of the congress come ideas and strategies that form the emerging discipline of mathematics in the classroom.

Facilitating the Math Congress

Choose several posters reflecting different approaches to reading the dials and explain that since the use of the dials is a mystery, the important discussion is about how they are related and how they change. As students present their theories, they will need to justify their reasoning. An important issue will be whether students record the number closest to where the hand is pointing or if they round up or down.

Some students will record the dial readings by writing down the digit closest to the pointer. This issue should come up in the congress. The notion that the dials are some sort of a clock is useful, and during the investigation students might ask if this is what happens on a clock. How does the position of the hand determine the hour? This way they can consider the question of the relationship between the rotation of the hands on the dials and the tick marks.

A Portion of the Math Congress

John (the teacher): Edgar and Rhonda, tell us what you think the first dial reads.

Edgar: Well, I thought the first one read 36.061, but Rhonda disagreed. She thought it read 35.961.

Rhonda: See, on this third dial, it looks like it’s on the 0, but it’s not quite there yet. I was thinking that they work like clocks, so since it’s not quite on the 0 yet, it should count as a 5.

Edgar: I still think it’s a 0, though.

John: Rhonda, can you maybe explain more?

Rhonda: Like on a clock, if it was 2:10, then the hour hand would be between the 2 and the 3. Since it’s not quite on the 3 yet, the hour is 2.

Edgar: But there’s no minute hand.

John: Hmm, that is confusing, isn’t it? It seems we have some questions about what to do here. Sasha, you and Carmen started off recording in a different way. Would you share next? Tell us what you did.

Sasha: We saw that the dials had smaller marks between the numbers, so we looked at where the hand was pointing. In the first dial, it’s past the 3 and on the fifth mark, so we recorded the first dial as 35.

Carmen: We kept doing that and got 35, 59, 96, 61, 10.

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Underscoring our ongoing and authentic approach to evaluation, regular assessment tips are provided where and when you need them.

Each day ends with reflections on the big ideas and strategies students explored during the workshop.
Appendix C  Student recording sheet for the weird dials investigation

Names ___________________________________________ Date _______________________

Zig’s notes:

I noticed that the hands move. I will watch really carefully and every ten minutes I will draw a picture and record what the dials show.

Here are my results. Wow! I think I am beginning to see how the dials are related.

What do you think Zig means? How do you think the hands on the dials move? If you were to write down numbers for what you see, what would you write?

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Teaching tools like this student recording sheet appear in a reproducible format in the back of each unit. They are also provided in an easy-to-access PDF format on the Teaching Resources CD-ROM.