Field Trips and Fund-Raisers

Introducing Fractions

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Contents

DAY ONE
The Field Trip

Today, the context of fair-sharing submarine sandwiches is introduced to support the development of several big ideas related to fractions. Students explore four situations: 3 subs shared by 4 people, 4 subs shared by 5 people, 7 subs shared by 8 people, and 3 subs shared by 5 people. Students work with partners to determine how much of a sub each person in each of these groups receives. They then make posters of some of the ideas they want to share in a math congress, to be held on Day Two.

Day One Outline

Developing the Context

- Tell the story about the distribution of submarine sandwiches on a field trip.
- Ask students to work in pairs to determine if the distribution was fair and to figure out what portion of a sandwich each person received.

Supporting the Investigation

- Note students' strategies as they investigate how much of a sub each person received.
- Ensure that they indicate where the subs should be cut.
- Support students as they move on to figure out which group received the most to eat.

Preparing for the Math Congress

- Have students make posters of the ideas and strategies they want to present during the math congress on Day Two.
- Review students' posters to determine who you will have share during the congress and what strategies you will highlight.

Materials Needed

Field trip posters [If you do not have the full-color posters (available from Heinemann), you can use the smaller black-and-white versions in Appendix A.]

Large chart paper—a few sheets per pair of students

Connecting cubes—one bin per pair of students

Calculators, as needed

Markers

Developing the Context

- Tell the story about the distribution of submarine sandwiches on a field trip.
- Ask students to work in pairs to determine if the distribution was fair and to figure out what portion of a sandwich each person received.

Display the four field trip posters (or Appendix A) as you tell the following story:

> *A fifth-grade class traveled on a field trip in four separate cars. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch, the subs were cut and shared as follows:*
>
> - *The first group had 4 people and shared 3 subs equally.*
> - *The second group had 5 people and shared 4 subs equally.*
> - *The third group had 8 people and shared 7 subs equally.*
> - *The last group had 5 people and shared 3 subs equally.*
>
> *When they returned from the field trip, the children began to argue that the distribution of sandwiches had not been fair, that some children got more to eat than the others. Were they right? Or did everyone get the same amount?*

Behind the Numbers

The numbers in this story have been chosen purposefully. Many students initially think that 3 subs for 4 people, 4 subs for 5 people, and 7 subs for 8 people are equivalent situations since there is always 1 sub fewer than the number of people. They will usually also argue that 3 subs for 4 people is not equivalent to 3 subs for 5 people, because more people are sharing when there are 5 and thus the pieces are smaller. When students go off to investigate, they will be surprised to discover that the first three situations are not equivalent. But they are correct about the fourth situation, and this beginning notion of fractions as division will be deepened as you proceed through the unit. Given these specific numbers in the context, students' conceptions at the start regarding proportional reasoning are illuminated. Don't try to dissuade them of their convictions at this point. Just facilitate a discussion and then let them work in pairs to figure out how much each person in each group received. Use the discussion as a motivator for inquiry and let them be surprised at the results.

Facilitate a preliminary discussion in the meeting area before the students set off to work. Allow students to share their initial thoughts and then ask them to work in pairs to investigate these questions:

1. Was the distribution fair—did each person in each group get the same amount?
2. How much of a sub did each person get, assuming the pieces were cut equally?

Supporting the Investigation

- Note students' strategies as they investigate how much of a sub each person received.
- Ensure that they indicate where the subs should be cut.
- Support students as they move on to figure out which group received the most to eat.

Assign math partners and give each pair of students some large chart paper. Have calculators and bins of connecting cubes available so that students may use them if they wish. Confer with students as needed to support and challenge.

Conferring with Students at Work

Inside One Classroom

Gabrielle: Three subs for 5 people. I think we should cut each sub in half first.

Michael: OK. *(Makes a line through each of three subs at the halfway point, producing six halves.)* But there are only 5 people. What do we do with this last half?

Gabrielle: Let's cut that up into 5 pieces. There. So everyone in this car got $\frac{1}{2} + \frac{1}{5}$. Now let's do 3 subs for 4.

Caroline (the teacher)**:** *(pointing to the small sliver)* Tell me about this piece.

Gabrielle: We cut it into 5 pieces so everyone could have a piece of what was left.

Caroline: I see that you wrote $\frac{1}{5}$ on your paper. Is this $\frac{1}{5}$ of a sub?

Gabrielle: Oh… No. It is $\frac{1}{5}$ of the half.

Caroline: Hmmm… $\frac{1}{5}$ of $\frac{1}{2}$? I wonder how much of the whole sub that is?

Author's Notes

Move around the room, noting the strategies being used. Confer with a few groups as they work.

Many students will start by cutting the sandwiches into large landmark pieces such as halves or thirds. When faced with the leftover pieces, they will then cut slivers and struggle with what to name those pieces.

By staying grounded in the context of the sandwich, Caroline encourages the students to realize that the size of the whole matters: $\frac{1}{5}$ of a half is not the same as $\frac{1}{5}$ of a sandwich.

By asking the students to reflect on how many slivers ($\frac{1}{5}$ of $\frac{1}{2}$) will fit in the whole sandwich, Caroline supports them to realize that if the other half were cut similarly there would be ten slivers. She helps them to realize that $\frac{1}{5}$ of $\frac{1}{2}$ is $\frac{1}{10}$.

Take note of the various struggles and strategies you see as students investigate how much of a sub each person got. Encourage struggles to become beautiful inquiries! As students cut up the subs, you might see them

- cut each sub into landmark fractions first, such as halves or thirds, and then cut the remainder into slivers. This strategy may cause them to struggle with what to name each piece: What do you call $\frac{1}{5}$ of a half? *[See Figure 1, page 16]*
- cut each sub into a number of pieces that is the same as the number of people. For example, when sharing 3 subs among 5 people, each of the 3 subs is cut into 5 pieces, resulting in $\frac{1}{5}$ of each sub per student, or $3 \times \frac{1}{5}$. This strategy may cause students to struggle with the notion that fractions are relations and that the size or amount of the whole matters. Everyone gets $\frac{3}{15}$ of the pieces, but this

Behind the Numbers

When they begin to compare and/or add pieces together, some students may attempt to represent subs with the connecting cubes, but may not make equal size subs. If you see students using various sizes, be sure to point this out by asking if one group received bigger subs. Stay grounded in the context to help students realize the meaning of what they are doing. As students struggle to ensure that the subs are all the same size, they will need to grapple with what numbers would be easy to use for the size of a common-length sub in order to compare and/or add the fractional amounts. Ten cubes may work nicely for fifths, but not so nicely for fourths or eighths! This inquiry will push students toward constructing the idea that a common multiple would be helpful. This idea is important for the construction of common denominators.

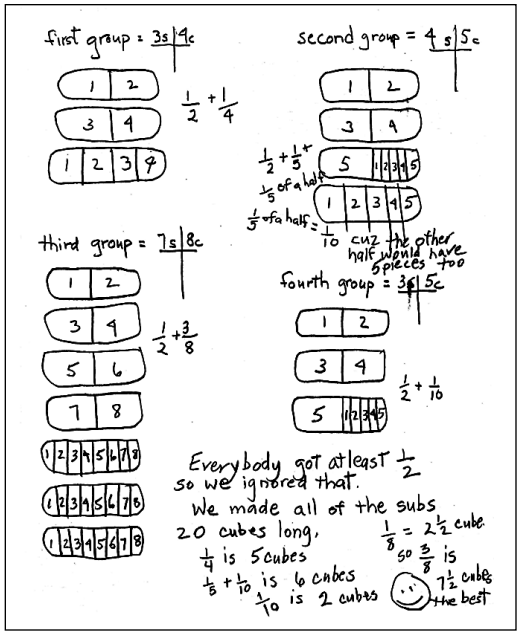

Figure 1

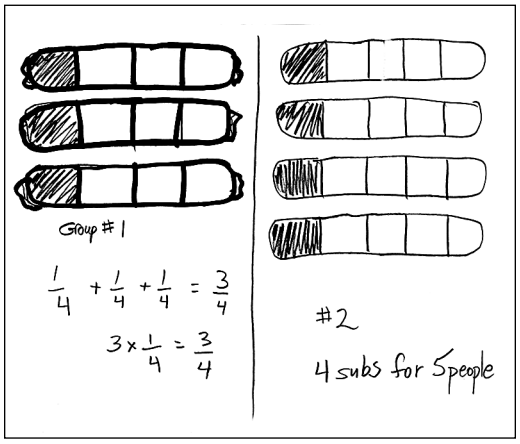

Figure 2

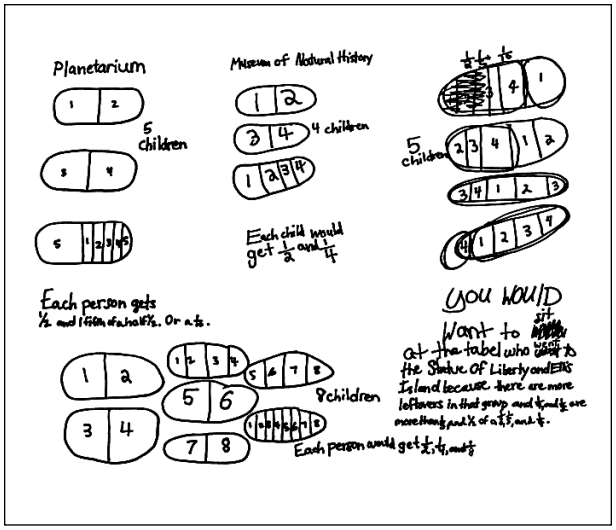

Figure 3

amount is also $\frac{3}{5}$ of one sub. Students may also notice that fractions can be thought of as division: 3 subs shared by 5 people results in $\frac{3}{5}$, or $3 \times \frac{1}{5}$. *[See Figure 2]*

- use the long division algorithm or a calculator to derive a decimal quotient: 3 divided by 5 equals 0.6. This strategy often prompts students to inquire about equivalence, "How is it that some students got $\frac{3}{5}$ and I got 0.6?" Encourage students to consider whether one answer is wrong or whether they are equivalent.

Ensure that all students indicate where the cuts would be made. If some students use a calculator to derive, for example, a quotient of 0.6 for 3 divided by 5, ask them to determine where the decimal equivalent would be on their drawing of the submarine sandwich. If they have not worked with decimals prior to this unit and they ask you what the decimal point means, you can tell them that 0.6 means six-tenths and then encourage them to think of the sandwiches in tenths.

Once students have cut up or shared the subs, they have to compare the results to determine which group got the most. Here you might see students

- work with unit fractions and ignore equivalent amounts:
  1. three subs for 4 people is $\frac{1}{2} + \frac{1}{4}$
  2. four subs for 5 people is $\frac{1}{2} + \frac{1}{5} + \frac{1}{10}$
  3. seven subs for 8 people is $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$
  4. three subs for 5 people is $\frac{1}{2} + \frac{1}{10}$

In each case everyone got at least $\frac{1}{2}$, so those halves can be ignored when comparing the situations. Group #3 received $\frac{1}{8}$ more per person than group #1, and group #2 received $\frac{1}{5}$ more per person than group #4. Hence, groups #1 and #4 can be ruled out and only groups #2 and #3 need to be compared to determine which of the four groups got the most. This strategy brings students nicely to examining denominators as divisors. One sub shared by 4 people results in larger pieces than 1 sub shared by 5 people! Similarly, $\frac{1}{8}$ is a larger piece than $\frac{1}{10}$. When comparing unit fractions, the greater the denominator, the smaller the piece is. *[See Figure 3]*

- compare common fractions or decimal equivalents by finding a common whole.

If students have divided each sub by the number of people in the group, they have obtained $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, and $\frac{3}{5}$ (or decimal equivalents) and now they have to compare those fractions. This strategy usually produces an inquiry about what size to make the subs so that they can be compared. At first, students may simply use trial and error, but eventually they will recognize that it is easier to use a common multiple of the number of people (a common denominator). *[See Figure 4]*

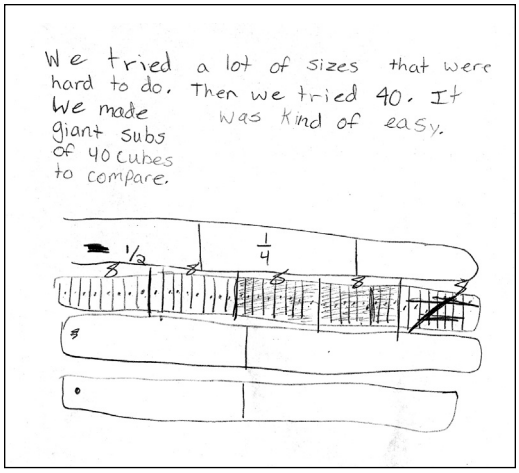

Figure 4

Preparing for the Math Congress

After a sufficient amount of time has been devoted to the investigation, ask students to make posters in preparation for the math congress to be held on Day Two. Explain that the posters need to be clear for others to understand. They should not be just students' draft notes copied over. They should be concise and clear presentations of the important ideas and strategies students want to present.

- Have students make posters of the ideas and strategies they want to present during the math congress on Day Two.
- Review students' posters to determine who you will have share during the congress and what strategies you will highlight.

Mathematicians write up their mathematics for math journals. In these articles, they do not merely reiterate everything they did. Instead, they craft a proof or argument for other mathematicians. Doing this not only generates further reflection, it focuses the author on developing a convincing and elegant argument—an important part of mathematics. Of course, elementary students are not expected to write formal proofs, but by focusing on the justification and logic of their arguments you are helping them develop the ability to write up their ideas for presentation in a mathematical community. For example, if students have constructed the idea that fractions are division, that 3 subs for 5 children is $\frac{3}{5}$ of a sub for each child, push them to generalize the idea and suggest that they focus their poster on proving this generalization.

Tips for Structuring the Math Congress

Plan on structuring a congress to discuss some of the big ideas related to fractions. Examine the posters as you plan for the congress and think about how you want the conversation to flow. By noting students' struggles and strategies, you can make a decision regarding which students you will ask to share and the strategies you will highlight. You will want to be sure to discuss the idea that fractions can be thought of as division: 3 subs shared by 5 people results in $\frac{3}{5}$ of a sub per person.

Usually, some student work will also illustrate a unit fraction strategy. Highlighting such a strategy will generate discussion of big ideas such as the following:

- the size or amount of the whole matters
- with unit fractions, the greater the denominator, the smaller the piece is
- when naming the piece, the whole matters (what to call $\frac{1}{5}$ of $\frac{1}{2}$, etc.)

Reflections on the Day

Several big ideas about fractions were explored today as students investigated the fair-sharing situation. Some groups grappled with how to compare the amounts and developed a beginning understanding of common denominators. Others grappled with equivalence, for example, what to call $\frac{1}{5}$ of $\frac{1}{2}$. Some realized that with unit fractions, the greater the denominator, the smaller the piece is. Still others, using a division strategy, determined that 3 subs shared by 5 people results in $\frac{3}{5}$ of one sub per person and they worked to generalize this idea.