Investigating Fractions, Decimals, and Percents

Unit Overviews for Grades 4–6

Investigating Fractions, Decimals, and Percents (Grades 4–6) is organized around 5 units. Each unit is developed around carefully-crafted contexts—realistic and fictional— and comprises a two-week (10 days) sequence of investigations, games, routines, and minilessons.

1 Field Trips and Fund-Raisers: Introducing Fractions

by Catherine Twomey Fosnot

The focus of this unit is the development of fractions. It begins with the story of a class field trip. The class is split into four groups and each group is given submarine sandwiches to share for lunch. Upon returning from their trip, the students quarrel over whether some received more to eat than others.

Note: This unit begins with the fair sharing of submarine sandwiches on a field trip. This context was field-tested by the Freudenthal Institute and the University of Wisconsin, under the direction of Thomas Romberg and Jan de Lange, in preparation for the writing of *Mathematics in Context: Some of the Parts* (van Galen, Wijers, Burrill, and Spence 1997) and it has been researched and written about extensively as it is used in this unit by Fosnot and Dolk (2002).

This story sets the stage for a series of investigations. First, students investigate whether the situation in the story was fair—was the quarreling justified?—thereby exploring the connection between division and fractions, as well as ways to compare fractional amounts. As the unit progresses, students explore other cases to determine fair sharing and then make a ratio table to ensure fair sharing during their future field trips. They also design a 60k bike course for a fund-raiser, a context that introduces a bar model for fractions and provides students with another opportunity to explore equivalent fractions.

Several minilessons for division of whole numbers using simplified equivalents are also included in the unit. These are structured using strings of related problems as a way to more explicitly guide learners toward computational fluency with whole-number division and to build a connection to equivalent fractions.

Note: The context for this unit assumes that your students have had prior experience with arrays for multiplication and division, as well as partitive and quotative division with whole numbers. If this is not the case, you might find it helpful to first use *The Teachers' Lounge* and *Minilessons Throughout the Year: Multiplication and Division* from *Investigations in Multiplication and Division:* Grades 3–5.

2 The California Frog-Jumping Contest: Algebra

by Bill Jacob and Catherine Twomey Fosnot

This unit uses the context of the famous short story by Mark Twain—*The Celebrated Jumping Frog of Calaveras County*—to develop equivalence and its use in solving algebraic problems. The context of a frog jumping along a track is used to foster number line representations in which students solve for an unknown amount, which is usually the length of a frog jump. Equivalent sequences of jumps are represented naturally on a double number line by having them start and end at the same location, with one expression shown on top of the line and the other shown underneath the line. The representation can then be used as a tool for solving the problem.

The unit begins with a problem in which students find the length of a bullfrog's jump, knowing the full length of a sequence of his jumps and steps. This context leads to using the number line as a tool for solving problems with unknowns. Next, students must find various approaches for lining up six- or eight-foot benches for two jumping tracks of lengths 28 and 42 feet. Students utilize the equivalence $6 + 6 + 6 + 6 = 8 + 8 + 8$ to change one possible solution into a second possible solution and use the number line to represent this equivalence. A similar problem about fences is used to develop a combination chart, which is a useful representation for determining net gain (or loss) after an exchange.

The second half of the unit includes more frog-jumping problems as the frogs plan for their Olympic Games. Now students further explore the use of variables to represent more complex situations and solve for unknown amounts. Here, students use the number line to represent jumps in the problems and can separate off equal amounts of unknown lengths to determine the lengths of unknown amounts. As the unit progresses, the questions require that students investigate equivalent lengths of different-sized jumps and work with these equivalences flexibly to solve problems.

The complexity of learning to symbolize has been the subject of extensive research. One study, summarized in *Adding It Up* (National Research Council 2001, 264), illustrates typical difficulties students may have. Known as the reversal error, it is illustrated by work on the following problem: At a certain university, there are six times as many students as professors. Using S for the number of students and P for the number of professors, write an equation that gives the relation between the number of students and the number of professors. A majority of students, ranging from first-year algebra students to college freshmen, wrote the equation $6S = P$. Apparently they used 6 as an adjective and S as a noun, following the natural language in the problem. However, they needed to multiply the number of professors by 6 to find the number of students. The correct response is $6P = S$. Because learning to write algebraic expressions is so difficult, we don't push symbolizing early in this unit. The representation of the number line is used to fix students' attention on the distinction between the lengths of jumps and the number of jumps. Once this is set, students can begin symbolizing in problems like this in a meaningful way. The unit ends with the students constructing more formal algebraic notation as they develop methods to simplify their earlier representations.

3 Best Buys, Ratios, and Rates: Addition and Subtraction of Fractions

BY BILL JACOB AND CATHERINE TWOMEY FOSNOT

The focus of this unit is the development of equivalence of fractions, proportional reasoning, and rates. It begins with a comparison of the cost of cat food at two stores: Bob's Best Buys where it is on sale, \$15 for 12 cans, and Maria's Pet Emporium where it is on sale, \$23 for 20 cans. Several important ideas and representations develop as students explore this problem, among them finding ways to determine the cost of a common numbers of cans for comparison and the use of the ratio table to represent their proportional reasoning about the context. The development of the ratio table is further supported in the next investigation as students work to determine the cost of several different amounts of bird seed sold by weight. As the unit progresses, proportional reasoning is once again the focus as students develop recipes for a variety of containers, using the recipe of Maria's gourmet puppy snack mix.

In the second week the double number is introduced for computation as students investigate the readings on a farm truck's gas tank over the course of trips to several neighboring farms to pick up produce. A trip across the Pennsylvania Turnpike is also explored.

This unit also includes several minilessons for addition and subtraction of fractions. Strings of related problems are used initially using money and clock models. Double number lines are introduced later in the unit to enable students to develop generalizable strategies for addition and subtraction. This model supports students to choose a common multiple (or factor) to work with as well as further opportunities to explore equivalent fractions.

Note: The context for this unit assumes that your students have had prior experience with fractions and their relationship to division with whole numbers. If this is not the case, you might find it helpful to first use the units *Field Trips and Fund-Raisers.*

4 The Mystery of the Meters: Decimals

by Bill Jacob, John Michael Siegfried, and Catherine Twomey Fosnot

This unit begins with the story of Zig—who discovers five mysterious dials on the side of his house. The dials are part of the electric meter for his house. At first Zig does not know this and he sets out to investigate how the dials work. As he collects data (readings every ten minutes), he notices that the hands on the dials turn in relation to each other (since each dial represents a different power of ten). Using Zig's data, students investigate how the dials are related. As the unit progresses, students use readings from the meter to measure energy to the thousandth of a kilowatt-hour to calculate the amount of energy used during a specific time period, and to determine readings on missing or obscured dials, working with place value equivalents.

The unit focus is on decimals, and since the electric meter in this unit represents kilowatt-hours to the thousandths, it can be used as a model to represent decimals. Because students can see how the numbers expressed as decimals increase with time, the meter is a powerful tool for students to use to determine equivalents and to examine how decimals increase and are ordered.

The electric meter consists of five circular dials numbered zero through nine that are lined up in place value order. As the hand on each dial makes a complete revolution, the number indicated on the dial to its left increases by one (one-tenth of a revolution.) This model was chosen because the position of the dials supports understanding of place value with decimals in tenths, hundredths, and thousandths. In examining how the hands on the dials move as the values increase, students may have opportunities to confront basic cognitive obstacles in making sense of decimal representations. These dials advance in the same way that the mechanical odometers of old cars worked, so exploring this mechanism provides an experience that students don't get these days due to the use of electronic digital odometers.

The electric meter used in this unit measures thousandths of a kilowatt-hour (or watt-hours), while the meters in most homes measure kilowatt-hours. This change was made to enable students to interpret the action of the meter in operating everyday electric devices, such as a refrigerator, light bulb, or television. When students go home and look at their meters, they may discover this as well as other similarities and differences. You can explain that in the story the meter is different, and you can invite students to think about the relationship of the problems in this unit to the meter they have at home.

5 Exploring Parks and Playgrounds: Multiplication and Division of Fractions

by Lynn D. Tarlow and Catherine Twomey Fosnot

The focus of this unit is the development of students' understanding of multiplication with rational numbers. The context of parks and playgrounds is used to introduce the double number line and the array models as helpful tools. Initially students are asked to explore the situation of two cousins running on a 26-mile course that winds through a park. They each ran half the course the previous year, and this year they complete 7/12 and 5/8 of the course, respectively. They know the exact portions they ran because of the course markers and water stations along the course. Students work to determine the number of miles that the cousins completed this year. In this first investigation, the distributive property and the use of landmark fractions (such as 1/2) provide a focus.

Next students work with the training data of a running club whose members train on a three-mile track in the park. This context is used to engage students in exploring the relationship between the operations of multiplication and division with rational numbers. Many students confuse these operations when working with fractions, so they are deliberately juxtaposed at the beginning of the unit as a way to prevent misconceptions from developing.

As the unit progresses into the second week, the context of designing a playground is used to support the development of specific mathematical ideas about the commutative property of multiplication with rational numbers, about the use of the word "of" to denote multiplication, and about fractional parts in relation to a changing whole. Students explore the story of two different empty lots: a section of each lot will be used as a playground, and then part of each playground will be covered with blacktop to be used for games like basketball and kickball. Both lots are the same size, but the part allocated to be a playground and the part to be blacktopped differ for each. The students investigate the problem of determining whether one lot has more blacktop, exploring equivalent, but not necessarily, congruent areas. The playground context supports students' use of an array model for multiplication, and the focus shifts to the commutative and associative properties.

Several minilessons for multiplication of rational numbers are also included in the unit. These are structured using strings of related problems as a way to explicitly guide learners toward computational fluency.