

CONTEXTS FOR LEARNING
mathematics

by **Catherine Twomey Fosnot** and **Colleagues**
from Mathematics in the City and the Freudenthal Institute



Investigating
**Number Sense, Addition,
and Subtraction**

(GRADES K–3)

Investigating
Multiplication and Division

(GRADES 3–5)

Investigating
Fractions, Decimals, and Percents

(GRADES 4–6)



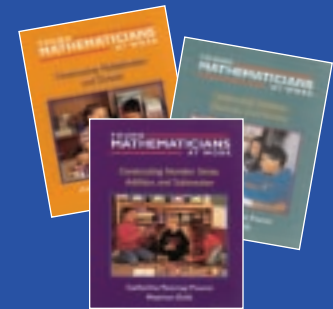
CONTEXTSFORLEARNING.COM



The Authors

☀ Catherine Twomey Fosnot

- Professor of Education at the City College of New York
- Director of Mathematics in the City
- Author of books and articles on mathematics education including the *Young Mathematicians at Work* series



☀ Colleagues from Mathematics in the City

- Located at City College of New York
- National inservice provider for mathematics educators, K–8

☀ Colleagues from Freudenthal Institute

- Located at Utrecht University, the Netherlands
- Researches and promotes best practices in math education



Overview

- ☀ Carefully crafted math situations foster a deep conceptual understanding of mathematical ideas, strategies, and models
- ☀ Investigations build on the ideals and structures of a math workshop
- ☀ Units model the teaching moves and language of master teachers
- ☀ Used as supplemental units or replacement units



Overview

“The rich, open investigations we’ve developed allow children to engage in mathematizing in a variety of ways. We honor children’s initial attempts at structuring and modeling their world mathematically, while at the same time supporting and challenging them to ensure that important big ideas and strategies are being developed progressively.... Learning, real learning, is messy; it’s not linear. We conceive of learning as a developmental journey along a landscape of learning.”

—CATHERINE TWOMEY FOSNOT



Organization

☀ Carefully Crafted Contexts

- Read-aloud books and posters

☀ Unit Books

- 2-week sequences of investigations, minilessons, and games

☀ Yearlong Resource Guides

- Strings of related games and minilessons

☀ Built-In Professional Support

- Overview
- Resources CD-ROM

☀ Additional Professional Development



Carefully Crafted Contexts

“Throughout this series we used contexts from children’s lives as starting points and crafted them to support learning. We built potentially realizable suggestions and constraints into the contexts and carefully chose numbers that would foster and support the development of specific strategies.”

—CATHERINE TWOMEY FOSNOT



Carefully Crafted Contexts

- ☀ **Set the stage for learning**
- ☀ **Developed through stories and pictures**
 - Read-aloud books (grades K–3)
 - Posters (grades 3–5 and 4–6)
- ☀ **Carefully crafted**
 - Show age-appropriate children using mathematics
 - Present imaginable situations—realistic and fictional
 - Are designed around landmark numbers and number relationships
 - Focus on meaningful storylines and metaphors



Carefully Crafted Contexts

In grades K–3 read-aloud books establish the contexts for learning

- ☀ Eight 16-page books
- ☀ Expansive 15" x 12" full-color format
- ☀ Reflect a range of topics and illustration styles





Carefully Crafted Contexts

In grades 3–5 and 4–6 posters establish the contexts for learning

- ☀ 17 posters in grades 3–5
- ☀ 16 posters in grades 4–6
- ☀ 15" x 24" full-color format





Carefully Crafted Contexts

Contexts include age-appropriate children using mathematics to solve real-world problems

Posters like these show groups of children discussing equitable ways to share an uneven number of sandwiches. This context sets the stage for learning about fractions.





Carefully Crafted Contexts

Contexts are developed around landmark numbers or number relationships that are significant and telling

The careful arrangement of items in these posters invite repeated addition, skip-counting, and doubling strategies, as well as introduce the language of grouping. These and other related posters lay the foundation for investigating fundamental multiplication strategies.



Carefully Crafted Contexts

The models and metaphors within a context make relationships and strategies more tangible and explicit

During the course of an investigation seemingly simple boxes of chocolate introduce students to the open array as a model for multiplication and division.





Unit Books

- ☀️ **Comprise a two-week (10-day) sequence of investigations, games, routines, and minilessons**
- ☀️ **Focus on big ideas, progressive strategies, and emerging models**
- ☀️ **Involve students in investigating, discussing, and constructing mathematical solutions**
- ☀️ **Encourage emergent learning and highlight the developmental landmarks in mathematical thinking**



Unit Books

Investigating **Number Sense, Addition, and Subtraction**

(GRADES K–3)

Eight unit books support the development of such fundamental topics as place value, compensation and equivalence, addition and subtraction on the open number line, and the efficient use of five- and ten-structures





Unit Books

Investigating **Multiplication and Division**

(GRADES 3–5)

Five unit books explore with increasing sophistication big ideas in multiplication and division including systematic factoring, the distributive, associative, and commutative properties as well as their use in computation.





Unit Books

Investigating
**Fractions, Decimals,
and Percents**

(GRADES 4–6)

Five unit books examine fundamental topics such as equivalence of fractions, operations with fractions, proportional reasoning, rates, and the ordering of decimals.





Unit Books

“The heart of the math workshop consists of ongoing investigations developed within contexts and situations that enable children to mathematize their lives. As children work, the teacher moves around the classroom, listening, conferring, supporting, challenging, and celebrating.”

—CATHERINE TWOMEY FOSNOT



Session Structure

Each unit book has a 10-day sequence of instruction. Each session is structured to support your math workshop.

- ☀ Introduction
- ☀ Brief Minilesson
- ☀ Developing the Context
- ☀ Supporting the Investigation
- ☀ Preparing for the Math Congress
- ☀ Facilitating the Math Congress



Session Structure

Introduction

- ☀️ **Locates session in the larger curriculum**
- ☀️ **Outlines the day's teaching moves**
- ☀️ **Lists necessary resources**

DAY THREE

Charts for the Grocer—Turkey

Today's math workshop begins with a warm-up minilesson on multiplication, using a string of related problems designed to encourage the use of the ratio table and facilitate the automatizing of the multiplication tables. Students then begin an investigation (using the ratio table) to produce charts that the grocer can display to help customers as they shop. Today students will work on a chart for turkey. On subsequent days they will work on charts for carrots, apples, and potatoes. The numbers on the charts have been carefully chosen to support students in moving away from repeated addition strategies toward more efficient grouping strategies.

Materials Needed

The turkey poster (or Appendix B)

Student recording sheet for the turkey chart (Appendix C)—one per pair of students

Before class, prepare an overhead transparency of Appendix C (or draw the t-chart on chart paper).

Overhead projector and overhead marker

Money—various amounts of quarters, dimes, nickels, and pennies

Large chart pad and easel (or chalkboard or whiteboard)

Markers

Day Three Outline

Minilesson: A Multiplication String

- ☀️ Work on a string of related problems designed to encourage students to use facts they know in order to figure out more difficult problems.
- ☀️ Record student strategies on a t-chart.

Developing the Context

- ☀️ Explain that students will develop charts that list the prices of turkeys of various sizes.
- ☀️ Do the first two problems in Appendix C as a whole group and then have students work with partners to complete the chart.

Supporting the Investigation

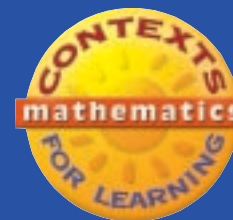
- ☀️ Encourage students to think about how the numbers in the chart might be helpful as they work.

Preparing for the Math Congress

- ☀️ Plan to focus the congress on the development of more efficient strategies.

Facilitating the Math Congress

- ☀️ Fill in Appendix C as students share and discuss their strategies.



Session Structure

Minilesson

☀ Brief (10-15 minutes)

☀ Jump starts the learning process

☀ Highlights computational strategies and involves students in mental math work

- ☀ Work on a string of related problems designed to encourage students to use facts they know in order to figure out more difficult problems.
- ☀ Record student strategies on a t-chart.

Minilesson: A Multiplication String (10–15 minutes)

The problems in this string have been chosen to encourage students to use facts they know as helper partial products in solving more difficult problems. The big idea underlying this strategy is the distributive property. The ratio table is a good model for illustrating this. It is helpful to represent the problems in a context, such as a number of cars and a number of tires, to help students realize the meaning of what they are doing. It is assumed that students working on this unit have not automatized the multiplication facts yet, and the strings in this unit are designed to help them do so by focusing on relationships.

Behind the Numbers

The first problem in this string will probably be easy for your students. Even if they don't know the answer automatically, they can use addition to figure it out. The first product can now be used to solve the second problem. Some students may still need to think of this as repeated addition, $4 + 4 + 4 + 4$. Some may even still need to use their fingers. The string is designed to encourage the use of more efficient strategies, so ask students to share a few strategies. If no one has thought to use the first problem to solve the second, encourage them to consider how it might be helpful—how $4 \times 4 = 2 \times 2 \times 4 = 2 \times 8$. The third problem may be more difficult, but the product of the second problem can now be used to help solve it. The fourth problem may be easy if students know the pattern of multiplication by ten. If a student says, "I just added a zero," encourage the class to recognize that putting a zero down is not adding, since $4 + 0 = 4$, not 40. Encourage them to realize that 10×4 is equivalent to 4×10 (which means that there are 4 tens; hence the zero is placed to bump the 4 over to the tens place). Other students may not know about the place value pattern that results when multiplying by ten, but they may think to use the third and first problems in the string. Adding the products of these problems produces ten groups of four, as well. Similarly, the next three problems can be solved by using others in the string. For example, 5×4 can be solved by halving the product of 10×4 . The last problem can be solved by adding another group of four to the product of the previous problem.

Number of Cars	Number of Tires
2	4
8	16

Do one problem at a time, giving students some think time before you start discussion. Record student strategies on a t-chart next to where you write the problem. Invite students to discuss the connection with their work on Day Two.

String of related problems:

- 2×4
- 4×4
- 8×4
- 10×4
- 9×4
- 12×4
- 5×4
- 6×4

■ Assessment Tips

It is helpful to keep a pad of sticky notes or some index cards near you when you do strings. If you can, jot down notes as you see students use interesting strategies and develop flexibility with the ratio table. Use one note or card for each student. At the end of the day, you can copy the string, attach your note, and place it in the student's portfolio. Make notes about the strategies and big ideas described in the introduction to this unit (pages 6–8).



Session Structure

Developing the Context

☀️ Introduces and enables children to realize the contexts for learning

Supporting the Investigation

☀️ Alerts you to strategies students are likely to exhibit

☀️ Suggests ways to confer with students as they work

Developing the Context

Begin by telling the students how amazed you were at all the wonderful strategies they used to determine the price of the 24-pound turkey. Suggest that it might be helpful if grocers posted charts to let customers know what their purchases would cost and invite students to make some charts with you. Use the overhead transparency of the turkey chart to demonstrate what you have in mind. Do the first two problems (two pounds and four pounds) with the whole group and then have students work in pairs to solve the remaining problems. Pass out one recording sheet (Appendix C) to each pair of students and have coins available should students need to use them.

- ☀️ Explain that students will develop charts that list the prices of turkeys of various sizes.
- ☀️ Do the first two problems in Appendix C as a whole group and then have students work with partners to complete the chart.

Appendix C

Number of Pounds	Cost
1	\$1.25
2	
4	\$4.00
8	
24	

Supporting the Investigation

As students work, walk around and take note of the strategies you see. Confer as needed to support and challenge their investigation. Do not tell students what strategy to use, but do encourage them to reflect on how the information on the chart might become helpful as they work. Remember to work with the mathematician, instead of trying to fix the mathematics! Focus not on the answer but on the thinking—on the strategies your students are trying out. Support students to think creatively and flexibly by encouraging them to think about how they might use one problem to help with another.

- ☀️ Encourage students to think about how the numbers in the chart might be helpful as they work.

It is assumed that one day is sufficient for students to complete this chart and to have a math congress on strategies they found helpful. Depending on their number sense, however, they may need more time. Some students may still need to use repeated addition strategies, thus taking much longer to solve the problems. It may be only after solving the problems that they notice patterns in their answers.

Behind the Numbers

The recording sheet is designed as a t-chart with missing numbers in order to encourage the use of the ratio table and grouping strategies based on distributivity. The numbers on the chart have been chosen carefully. For example, \$10.00 can be figured out by repeated addition until 10 is reached, or the 4 pounds for \$5.00 can be doubled. Ten pounds can also be solved by adding the cost of 2 pounds and 8 pounds, etc.



Session Structure

Preparing for the Math Congress

☀ Suggests strategies for helping investigation teams organize and present their findings

☀ Offers tips for structuring powerful discussions

Conferring with Students at Work



Author's Notes

(Diana (the teacher) joins a group of students but at first just listens to their conversation as they work.)

Diana begins by just listening.

Holly (Talking to Carolina): Four pounds was easy because we just doubled the 2 pounds. But what do we do with the tens? Why is that 10 over there (pointing to the 10 dollars) and this 10 is here (pointing to the 10 pounds)? Ten is there twice.

Carolina: Yeah. That's weird.

Diana: Hmm...that's interesting. What does each mean? Since they are in a different place on the chart, I wonder if they mean something different. What else does it say on the chart?

Rather than answering their question directly, Diana wonders aloud with them, while encouraging them to consider what the numbers mean in context.

Carolina: Oh, I get it. That 10 is the money, and this 10 is the 10 pounds. (Points to headings on the T-chart.) See...it says here, pounds, and there...cost.

Developing students' ability to read charts and tables is as important as calculation.

Holly: But I don't get it. Don't you have to know how many pounds to figure out the cost?

Diana: Well, let's see. What do we know so far? One pound is \$1.25, 2 pounds is \$2.50, and 4 pounds is \$5.00. Hmm...4 pounds for \$5.00. I wonder if that would help us.

By using the pronouns we and us, instead of you, Diana implicitly places herself in the inquiry. She does not make the students feel that she knows the answer and that they have to figure out their answer—a power dynamic that can destroy the desire to inquire. Instead, she models the joy of puzzlement while encouraging them to continue pondering.

Holly: But this is \$10.00—oh, I get it! The money doubled. So double the pounds!

Carolina: Yeah!

Diana: That's a great strategy! Wow! What a good idea. I bet you could use that strategy to figure out \$20.00, even though it isn't on the chart.

The students now have a strategy they own and feel comfortable with, and Diana celebrates with them even while she challenges them to reflect further.

Holly: Yeah, I bet we could!

Carolina: Yeah. We're great!

Preparing for the Math Congress

☼ Plan to focus the congress on the development of more efficient strategies.

The math congress today will not require the making of large posters; you can use the overhead transparency of the turkey chart for this class discussion. You will want to focus on just a few of the strategies students are proud of and excited about sharing. Think about the many strategies you have seen your students using. Some students are probably still doing repeated addition, while others may already be using the ratio table very efficiently. Noting the strategies that students are using now will help you plan the discussion to continue supporting students who are still struggling.



Session Structure

Facilitating the Math Congress

- ☀ Describes strategies for implementing and evaluating whole-group discussions
- ☀ Highlights ways of fostering a community of mathematicians

Facilitating the Math Congress

Use the overhead transparency of the turkey chart and fill it in as students share their strategies. Start with the third problem (since you and the class did the first two problems together when you developed the context). Ask a student who used doubling to share (illustrated in Inside One Classroom, below). If some students have used repeated addition, make sure that you bring them into the conversation and help them understand the doubling strategy. You can do this by writing down the repeated addition on chart paper and circling the 2 groups of 4 pounds, marking them as \$5.00 each. Then move to the 10 pounds. Have a variety of strategies shared here, such as adding the costs of 8 pounds and 2 pounds, or figuring out the cost of 5 pounds (perhaps adding the costs of 4 pounds and 1 pound) and doubling it. Continue to find and circle the groups discussed in the repeated addition. Then discuss strategies for figuring out the cost of 20 pounds. Once the overhead chart is filled in, ask students to consider how customers might use it to figure out numbers not on the chart, such as 6 pounds or 19 pounds.

☀ Fill in Appendix C as students share and discuss their strategies.

A Portion of the Math Congress



Author's Notes

Diana (the teacher): So now we've filled in the chart, but I'm wondering about customers who want to buy a turkey that's not listed on the chart, like a turkey that weighs 9 pounds, or 6, or 19. Turn to the person next to you and discuss this for a few minutes. Could customers use our chart to figure out the cost of other turkeys? (After several minutes of pair talk and listening in to a few conversations, Diana starts discussion.) Vicki? What did you and Carlos decide?

Pair talk provides important thinking and reflection time. Starting discussion on this important question prematurely would most likely result in only a few voices. Pair talk ensures that all students are involved.

Vicki: Yep. We decided they could! Six pounds you could do by adding the 2 and the 4. That's \$2.50 plus \$5.00.

Carlos: And 5 pounds is just \$1.25 plus \$5.00.

Diana: What about 9 pounds? Noreen?

Noreen: That is just \$1.25 less than the 10 pounds.

Diana: Wow! It seems we made a great chart! Could it be used for all of the numbers up to 24?

At the heart of doing mathematics is generalizing. Asking students to consider all the cases pushes them to generalize.



Instructional Features

Key instructional features help you get the most out of each session.

☀ **Step-by-step teaching advice**

☀ **Behind the Numbers**

☀ **Side-column teaching moves**

☀ **Inside One Classroom**

☀ **Assessment Tips**

☀ **Appendixes**



Instructional Features

☀️ Side-column notes outline **teaching moves**

☀️ Main column contains **step-by-step teaching advice** and professional insights

☀️ **Assessment Tips** support ongoing assessment and evaluation

☀️ Work on a string of related problems designed to encourage students to use facts they know in order to figure out more difficult problems.

☀️ Record student strategies on a t-chart.

Minilesson: A Multiplication String (10–15 minutes)

The problems in this string have been chosen to encourage students to use facts they know as helper partial products in solving more difficult problems. The big idea underlying this strategy is the distributive property. The ratio table is a good model for illustrating this. It is helpful to represent the problems in a context, such as a number of cars and a number of tires, to help students realize the meaning of what they are doing. It is assumed that students working on this unit have not automatized the multiplication facts yet, and the strings in this unit are designed to help them do so by focusing on relationships.

Behind the Numbers

The first problem in this string will probably be easy for your students. Even if they don't know the answer automatically, they can use addition to figure it out. The first product can now be used to solve the second problem. Some students may still need to think of this as repeated addition, $4 + 4 + 4 + 4$. Some may even still need to use their fingers. The string is designed to encourage the use of more efficient strategies, so ask students to share a few strategies. If no one has thought to use the first problem to solve the second, encourage them to consider how it might be helpful—how $4 \times 4 = 2 \times 2 \times 4 = 2 \times 8$. The third problem may be more difficult, but the product of the second problem can now be used to help solve it. The fourth problem may be easy if students know the pattern of multiplication by ten. If a student says, "I just added a zero," encourage the class to recognize that putting a zero down is not adding, since $4 + 0 = 4$, not 40. Encourage them to realize that 10×4 is equivalent to 4×10 (which means that there are 4 tens; hence the zero is placed to bump the 4 over to the tens place). Other students may not know about the place value pattern that results when multiplying by ten, but they may think to use the third and first problems in the string. Adding the products of these problems produces ten groups of four, as well. Similarly, the next three problems can be solved by using others in the string. For example, 5×4 can be solved by halving the product of 10×4 . The last problem can be solved by adding another group of four to the product of the previous problem.

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Instructional Features

Behind the Numbers explains

- ☀ **the significance of the numbers chosen**
- ☀ **why numbers are ordered the way they are**
- ☀ **how numbers work with the contexts to support the development of certain strategies, models, and big ideas.**

Behind the Numbers

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Instructional Features

Inside One Classroom provides a glimpse into a classroom community

☀️ **Dialogue excerpts** model teaching language and help you envision typical classroom discussions

☀️ **Author Notes** allow Cathy to team teach with you and share her experience and insights

Conferring with Students at Work



Inside One Classroom

(Diana (the teacher) joins a group of students but at first just listens to their conversation as they work.)

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Instructional Features

The **Appendixes** at the end of each unit provide:

- ☀ the context-setting posters and read-aloud books in a reproducible format
- ☀ each session's teaching tools including game boards, recording sheets, and templates

Appendix B

Turkey for
\$1.25 per p

Appendix C
Student recording sheet for the turkey chart

Names _____ Date _____

Number of Pounds	Cost
1	\$1.25
2	
4	
10	\$10
20	

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Resource Guides

- ☀ Games (K–1) and minilessons (1–6)
- ☀ Used throughout the year at the start of a math workshop
- ☀ Tightly structured minilessons help generate discussions on certain strategies and big ideas
- ☀ Strings of related problems develop deep number sense and expand a repertoire of strategies

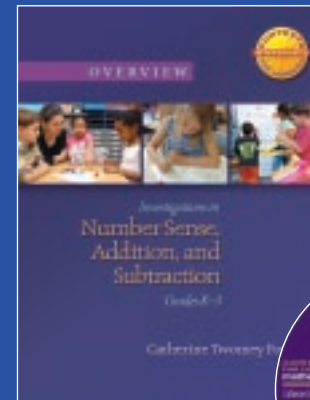




Built-In Professional Support

Overview

- ☀ Explains series' philosophy and components
- ☀ Outlines contents of each unit and suggests sequence of instruction



Resources CD-ROM

- ☀ print and video resources support math workshops



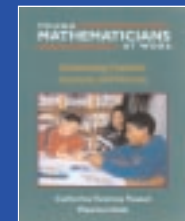
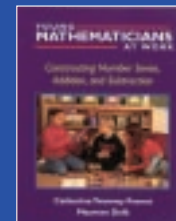


Additional Professional Development

Young Mathematicians at Work

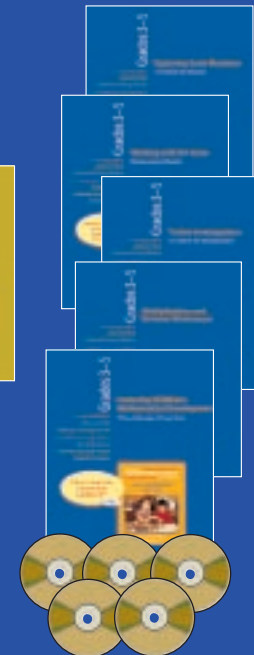
☀ Professional book series

- 3-volume series for teachers
- research and theories that underlie the development of students' understanding of number s and operations



☀ Professional Development Packages

- 5 interactive CD-ROMs per level show classroom clips of real teaching
- Overview manual & Facilitator's Guide



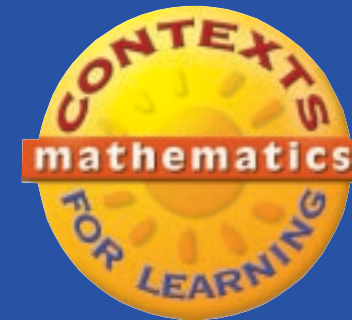


Additional Professional Support

Related inservice and workshops

- ☀ Mathematics in the City
www.mitccny.org
- ☀ Heinemann Professional Development
www.heinemann.pd.com
- ☀ On-site offerings and on-line courses
from Cathy and her colleagues
www.newperspectivesonlearning.com

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mathematics



by **Catherine Twomey Fosnot** and **Colleagues**
from Mathematics in the City and the Freudenthal Institute

**Every Student Can Be
a Mathematician!**

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